



## CREATING INTUITIVE TEACHING METHODS FOR ADVANCED MATHEMATICS EDUCATION: ORIGINAL APPROACHES TO SIMPLIFY SUCCESSIVE DIFFERENTIATION

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### Abstract

This study strives to transform advanced mathematics education through the introduction of original and intuitive teaching methods for successive differentiation. Using innovative methods, the goal of this research to simplify the complex situation of this concept, through which learners ensure better associability. This research explores unique pedagogical techniques and strategies which focusing on improving the clarity and understanding. In this paper we are discussing common difficulties in understanding successive differentiation; the aim proposed methods is to improve the whole learning experience. This research combines theoretical understandings with real world applications, which promotes a deeper understanding of successive differentiation. The aim of this research is to give valuable methods for teachers or educators and students, contributing to a more engaging and effective learning environment in the field of advanced mathematics.

**Keywords :** Intuitive teaching methods, advanced mathematics, successive differentiation.

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### Introduction

In the field of advanced mathematics education, this study presents a ground breaking initiative centred on idea of intuitive or innovative teaching methods for the concept of successive differentiation. The objective is to change the traditional and old approach, making this complex mathematical idea more accessible and understandable for students. [Fischbein (1987), Giant (2001), Rösken & Rolka (2007)]. By implementing of innovative and creative teaching techniques, the aim of this research is to simplify the difficulties associated with successive differentiation. By addressing the common difficulties in understanding this mathematical concept, the proposed methods are designed to significantly improve the whole educational experience. The study looks into the development of original methodologies and creates unique technique, maintaining transparency and developing a deep understanding of successive differentiation. This research combines theoretical understandings with real world applications to provide a comprehensive view which improves learning experience.

This research aims to offer useful tools for both educators and students with a dedication to bridging the gap between theory and real world applications, (Tall, 1991). The main goal of this research is to establish a more effective learning environment to engage the educators and students so that it can be catalyze a shift in such a way that successive differentiation is taught and absorbed in advanced mathematics education. Our aim is to establish the foundation for a revolutionary learning experience in advanced mathematics by this innovative research.

### Literature Review

The process of differentiating a function more than one times with respect to the same variable is called Successive differentiation. Its main objective is to analyse higher-order derivatives in order provide better understandings of the behaviour of the functions through which one can obtain information on slope, concavity, and point of inflexion and can aid up the understanding of changing rates. This is helpful in much discipline such as physics, engineering, and related fields where deep study of system or function behaviour is more important. The rate of change of the (n-1)th derivative is given mathematically by nth derivative. The applications of successive differentiation include solving differential equations through variety of scientific and engineering contexts, optimization, Taylor series expansions, and across

### Performances of Previous Methods

The method of successive differentiation also known as repeated differentiation more than one time that is a mathematical technique or method which is used for determining higher-order derivatives terms of a function. Its effectiveness depends on the complexity of the function and the degree of the differentiations applied. [Hadamard (1945)]. We are giving some points to modify performance of teaching method on successive differentiation:

**Precision:** By applying intuitive teaching method to determine higher order differentiations, the application of successive differentiation can give more precise and accurate results for standard functions [Giant D (2001)].

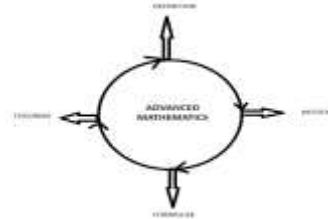
2. Computational Intensity: When we work with those functions which have more complex expressions, particularly those function that have time consuming and large manual calculations computational intensity becomes more beneficial [ Fujita *et al.* (2004)].
3. Effectiveness: Successive differentiation may be much more effective than any other methods or technique.[ H Poincaré (1913)]. For the calculation of higher order derivatives in some examples giving a simple and intuitive methods is very effective for handling standard functions [RR Skemp (1079)].
4. Applicability: This method is more applicable and reliable for those functions which contain polynomials, exponentials, and trigonometric, logarithmic, inverse trigonometric, hyperbolic and inverse hyperbolic trigonometric function because it may identify the pattern of successive differentiations [Q. Zerry (2010)].
5. Limits: Limit method is used in some functions or expressions which contain indeterminate (undefined) and discontinuous points their effectiveness (intensity) decrease because process may give undefined result.
6. Algorithmic Nature: Successive differentiation ( $y_n$ ) can give a methodical and algorithmic (analytical) nature which provide an Algorithmic process that help in understanding and improvement the nature of successive differentiation.
7. Learning Tool: It works as a learning tool to help the students and can give the ideas for higher derivatives in enhancing the relation or connection between a function and its successive differentiation [Noddings N (1985)].
8. Consolidation: This method may exhibit quick consolidation for some functions, facilitating the fast determination of higher-order successive differentiation, while for others, this may be slower [Fischbein E (1987)].
9. Automation: With the continuity of computational (determination) techniques and tools [Watson J (2000)] with the help of computers, the manual application of successive differentiation has become less because repetitive calculations are managed by software quickly.
10. Practicality: To maintain its importance as an analytical tool, practical applications often involve numerical methods or symbolic computation systems for determining higher-order differentiation in real-world problem-solving [Tall D (1980)].

### Objective of Study

The objective is to enhance comprehension and engagement in advanced mathematics by developing intuitive teaching methods. Original approaches aim to demystify successive differentiation, making it more accessible and fostering a deeper understanding among students.

### Methodology

- a. Attempt to comprehend the significance of the issue



- b.
- c. Now we have taken the topic of successive differentiation for solving in easier and innovative methods.

### Definition of Successive Differentiation

The process of differentiating the same function repeatedly is termed successive differentiation.

### Notation of Derivative

**Table 1: Notation**

St. No.							
1	First order of derivative	$\frac{dy}{dx}$	$Dy$	$y_1$	$y'$	$f'(x)$	
2	Second order of derivative	$\frac{d^2y}{dx^2}$	$D^2y$	$y_2$	$y''$	$f''(x)$	
3	Third order of derivative	$\frac{d^3y}{dx^3}$	$D^3y$	$y_3$	$y'''$	$f'''(x)$	
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
n	nth order of derivative	$\frac{d^n y}{dx^n}$	$D^n y$	$y_n$	$y^{(n)}$	$f^{(n)}(x)$	

### 3. Formulae

Function	$n^{th}$ Derivative
$y = e^{ax}$	$y_n = a^n e^{ax}$
$y = (ax + b)^m$	$y_n = m(m-1) \dots (m-n+1)n! a^n (ax+b)^{m-n}, m > 0, m > n$ $y_n = \frac{m!}{(m-n)!} a^n (ax+b)^{m-n}, m > 0, m > n$ $y_n = n! a^n, m = n$ $y_n = \frac{(-1)^n n! a^n}{(ax+b)^{n+1}}, m = -1$
$y = \log(ax + b)$	$y_n = \frac{(-1)^{n-1} (n-1)! a^n}{(ax+b)^n}$

$y = \sin(ax + b)$	$y_n = a^n \sin \left( ax + b + \frac{n\pi}{2} \right)$
$y = \cos(ax + b)$	$y_n = a^n \cos \left( ax + b + \frac{n\pi}{2} \right)$
$y = e^{ax} \sin(bx + c)$	$y_n = r^n e^{ax} \sin(bx + c + n\phi)$ Where $r = \sqrt{a^2 + b^2}$ and $\phi = \tan^{-1} \frac{b}{a}$
$y = e^{ax} \cos(bx + c)$	$y_n = r^n e^{ax} \sin(bx + c + n\phi)$ Where $r = \sqrt{a^2 + b^2}$ and $\phi = \tan^{-1} \frac{b}{a}$

**4. Theorems**

**a) Leibnitzs' Theorem**

This theorem is useful for finding the nth derivatives of product of two functions. This may written as follows:

If u and v be any two functions of x having differential coefficient nth order, then-

$$D^n(uv) = D^n u \cdot v + {}^n C_1 D^{n-1} u \cdot Dv + D^{n-2} u \cdot D^2 v + \dots + {}^n C_r D^{n-r} u \cdot D^r v + \dots + u \cdot D^n v$$

We shall prove this theorem by mathematical induction method. Assume that the above result is true for particular value of n. then differentiation with respect x, we have

$$D^{n+1}(uv) = \{ (D^{n+1} u) \cdot v + D^n u \cdot Dv \} + {}^n C_1 \{ D^n u \cdot Dv + D^{n-1} u \cdot D^2 v \} + \dots + {}^n C_r \{ D^{n-r+1} u \cdot D^r v + D^{n-r} u \cdot D^{r+1} v \} + \dots + \{ Du \cdot D^n v + u \cdot D^{n+1} v \}$$

Rearranging we have

$$D^{n+1}(uv) = (D^{n+1} u) \cdot v + (1 + {}^n C_1) (D^n u \cdot Dv) + \dots + ({}^n C_r + {}^{n+1} C_{r+1}) (D^{n-r} u \cdot D^{r+1} v) + \dots + u \cdot D^{n+1} v$$

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But

$${}^n C_r + {}^{n+1} C_{r+1} = {}^{n+1} C_{r+1}$$

Hence

$$D^{n+1}(uv) = (D^{n+1} u) \cdot v + {}^{n+1} C_1 D^n u \cdot Dv + \dots + {}^{n+1} C_{r+1} D^{n-r} u \cdot D^{r+1} v + \dots + u \cdot D^{n+1} v$$

Thus the theorem true for any value of n. hence we have the result.

**Results**

Applying these theory and techniques of successive differentiation we can make successive differentiation easy and effective. Creating these simple teaching methods in advanced mathematics we can easily simplify the successive differentiation.

**Conclusion and Future Scope**

Successive differentiation involves the repeated process of finding derivatives for a function. Its scope encompasses the analysis of higher-order derivatives, identification of inflection points, determination of concavity, and understanding the function's behaviour through various differentiation levels. This technique proves valuable in calculus, facilitating a detailed study of functions, their characteristics, and trends. Similarly using these intuitive teaching methods other topics of advanced mathematics can also be describe easily and effectively.

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